ADVECTION IN ACCRETIONDISCS BOUNDARY LAYERS

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ABSTRACT

Accretion disc boundary layer models have been calculated for various systems, using a one dimensional time dependent numerical code. At high accretion mass rates $\dot{M} \approx 10\text{--}4~M_{\odot}/y$, or low value of the viscosity parameter $\alpha \approx 0.001$ - 0.01, (characteristic of FU Orioni systems and some Symbiotic stars) the optical thickness in the inner part, of the disc becomes very large ($\tau >> 1$). The disc, unable to cool efficiently, becomes geometrically thick ($H/r \approx 0.5$), the energy dissipated in the dynamical boundary layer is radiated out ward to larger radii and advected into the star. The boundary layer luminosity is only a fraction of its expected Value, the rest of the energy is advected into the star. The fraction of the advected energy is $\zeta = L_{adv}/L_{acc} \approx 0.1$ in sybinbiotic stars (accretion onto a low-mass main-sequence star) and $\zeta \approx 0.2$ in FU Orioni systems (accretion onto a pre main-sequence star).

keywords: accretion, accretion discs - stars: pre-main-sequence - binaries: symbiotic methods: numerical

1. INTRODUCTION

In the standard theory of accretion discs (Shakura & Sunyaev 1973; Linden-Bell & Pringle 1974; Pringle 1981) half of the accretion energy $L_{acc} = GM_*\dot{M}/R_*$ is emitted from the disc, and the other half from the boundary layer. Since the radial extent of the boundary layer is very small $\delta_{BL} \approx \epsilon^2 r = H^2/r$ (where $\epsilon = H/r \approx c_s/v_K$), its temperature is expected to be very high. But, as pointed out by Pringle (1977), the energy dissipated in the boundary layer is emitted from a much wider region (the thermal boundary layer region of width δ_{BL}^{th}). It was only recently that numerical calculations were able to show the existence of the thermal boundary layer $\delta_{BL}^{th} \approx 2H$, with a temperature much lower than expected (Popham et al. 1993; Narayan & Popham 1993; Lioure & Le Contel 1994; Regev & Bertout 1995; Godon, Regev & Shaviv 1995).

Another important process in the theory of accretion discs, is advection of energy. A few studies were carried out on this subject in thick discs (Begelman & Meier 1982) and slim discs (A bra mowicz et al. 1988), and more recently in optically thin discs (Narayan & Yi 1994, 1995; Abramowicz et al. 1 995) and in boundary layers (Narayan & Popham 1 993; Pophamet al. 1993; Co clour 1996a & b). Advection becomes important when the cooling time of the accretion flow is longer than the radial infall time: the inflowing matter cannot cool efficiently and is accreted with its energy content. This situation arises when the optical thickness is very small (gas pressure dominated-flow with inefficient cooling) or very large (radiation pressure dominated-flow, where the radiation is trapped), or just when the radial in fall velocity is very high (transonic solutions). Consequently, advection naturally happens in accretion discs at either very low or very high accretion mass rate. Eventually, the advected energy reaches the inner boundary - the outer surface of the accreting object. If the accreting object is a black hole, then the advected energy is 'lost,' beyond the horizon. On the other hand, if the accreting object is a star, then the energy is advected into the outer stellar envelop. However, in this latter case, the energy advected into the star, was mainly dissipated in the boundary layer (Godon 199621 &b). Consequently, the boundary layer emission is missing in these systems, which is in fact what the observations show (FU Orionis, Kenyon, Hartmann & Hewett 1988; Symbiotic binaries AX Persei & CI Cygni, Kenyon et al. 1991, Mikołajewska & Kenyon 1992).

In the boundary layer region the radial infall velocity is large (it can even be supersonic, depending 011 the assumption on the viscosity) and the temperature is high (in comparison to the disc temperature). Therefore, one expects advection of energy to be non-negligible t 110°C. In extreme cases, when the boundary layer cannot cool efficiently, the advection of energy is actually the cooling process which tends to stabilize the inner part of' the disc (Abramowicz et al. 1995). Numerical simulations of accretion discs boundary layers (Narayan & Popham 1993; Popham et al. 1993) have shown the importance of advected entropy in the inner region of the discs in two extreme cases: when the optical depth is either very small ($\tau << 1$) or very large (7 >> 1). However, these results were mainly qualitative and no quantitative treatment of the advection in the boundary layer region has

been carried out so far. It is the purpose of the present work to carried out a quantitative treatment for the optically thick branch of solutions. We investigate two different kinds of optically thick solutions: boundary layers around accreting pre-main sequence stars (FU Orioni objects) and low-mass main sequence stars (Symbiotic binaries AX Per & CI Cyg). The first kind of solutions becomes advection-dominated for either large \dot{M} or small α ($\rho \approx \dot{M}/\alpha$), while the second kind of solutions is advection-dominated only for large \dot{M} . Both solutions have angular velocities substantially sub-Keplerian and $H/r \approx 0.5$. In fact these solutions do not represent thin discs any more, but rather advection dominated accretion flow as described by Narayan & Yi (1994, 1995).

In section 2 the numerical modeling of the boundary layer is reviewed. The results are presented and discussed in section 3.

2. ACCRETION DISCS BOUNDARY LAYERS MODELING

2.1 Equations and assumptions

The equations are written in cylindrical coordinates (r, ϕ, z) , and are vertically integrated. Assuming axi-symmetry, the remaining equations become one dimensional (in 7). They include the gravity of the accreting star, vise mits, and radiative transfer treated in the diffusion approximation. Radiation pressure is further taken into account in the equation of state. The medium is assumed to be optically thick. The exact form of tile equations and the physical assumptions have been widely described in Godon (1995), and Godon et al. (1995). Therefore, only a brief description of the assumptions are given here. For more details the reader is referred to the above references.

The equations are: the conservation of mass

$$\frac{\partial \Sigma}{\partial t} = -\frac{1}{r} \frac{\partial}{\partial r} (r \Sigma v_r), \tag{1}$$

the conservation of radial momentum

$$\frac{\partial}{\partial t}(v_r \Sigma) = -\frac{1}{r} \frac{\partial}{\partial r} [r v_r(v_r \Sigma)] - \{ r \Sigma \Omega^2 - \Sigma \frac{GM}{r^2} - \frac{\partial P}{\partial r} + F_{\nu},$$
 (2)

the conservation of angular momentum

$$\frac{\partial}{\partial t}(\Sigma r^2 \Omega) = -\frac{1}{r} \frac{\partial}{\partial r} (r \Sigma r^2 \Omega v_r) + \frac{1}{r} \frac{\partial}{\partial r} (r^3 \nu \Sigma \frac{\partial \Omega}{\partial r}), \tag{3}$$

and the conservation of energy

$$\Sigma T \frac{\partial S}{\partial t} = -\Sigma T v_r \frac{\partial S}{\partial r} + \nu \Sigma (r \frac{\partial \Omega}{\partial r})^2 - \frac{4acT^4}{3\kappa \Sigma} + \frac{16ac}{3r} \frac{\partial}{\partial r} \left[\frac{rH^2 T^3}{\kappa \Sigma} \frac{\partial T}{\partial r} \right]$$
(4)

where $\Sigma = \int_{-H(r)}^{+H(r)} \rho dz$, v_r is the radial velocity, Ω is the angular velocity, G is the gravity constant, M is the mass of the star, S is the entropy, a is the Stephan-Boltzmann constant, c is the speed of light, T is the mid-plane temperature, H is the half thickness of the disc and κ is the opacity. The pressure includes the gas and the radiation:

$$P = \frac{\Sigma \mathcal{R}T}{\mu} + \frac{1}{3}2HaT^4. \tag{5}$$

The radial viscous force F_{ν} is:

$$F_{\nu} = \frac{\partial}{\partial r} \left[\frac{4\nu\Sigma}{3r} \frac{\partial (rv_r)}{\partial r} \right] - 2 \frac{v_r}{r} \frac{\partial (\nu\Sigma)}{\partial r}, \tag{6}$$

where ν is the coefficient of the shear viscosity, \mathcal{R} is the gas constant and μ is the mean molecular weight.

In equations 2 we have kept a term of the order of v_r^2 (first term on the right hand side of the equation), since v_r can be large in the inner part of the disc. While in equations 4 (the energy equations) we have included the advected entropy (first term on the right hand side), the energy dissipated by viscous process Q^+ (the second term on the RHS), the leak of energy Q^- radiated in the vertical direction (third term on the RHS) and the radiation of energy in the radial direction (last term on the RHS). The above equations, used to represent the boundary layer (Regev 1983), were introduced by Paczyński & Bisnovatyi-Kogan (1981), and were later used by Abramowicz et al. (1988) as the slim disc equations (except for the pseudo-Newtonian potential, Paczyński & Wiita 1980).

At the inner boundary of the computational domain, the accreting star rotates at constant angular velocity $(\Omega_* = 0.1\Omega_K(R_*))$ and a constant inflow of matter (\dot{M}) flows into the stellar surface. A non-flux (dT/dr = 0) condition is also imposed at this boundary. At the outer boundary a geometrically thin Keplerian disc is assumed.

The method used in this work is a time-dependent method. The initial conditions are the superposition of an isothermal atmosphere and an inflowing disc of matter.

The viscosity prescription used here is similar to the one used in Godon (1996a). The value of the viscosity parameter varies from model to model.

2.2 The numerical method

The spatial dependence of the equations is treated with the use of a Chebyshev spectral method (Gottlieb & Orszag 1977; Voigt, Gottlieb & Hussaini 1984; Canutto et al. 1988), while the time dependence of the equations is solved with an implicit Crank-Nicholson scheme for the energy equation and an explicit fourth order Runge-Kutta method for the remaining equations. The time dependent Chebyshev pseudospectral method of collocation

has been developed recently to treat ast rophysical flows and has been applied to old-dimensional accretion disc boundary layers (Godon 1995, 1996a, & 1, Godon et al. 1995). All the details of the numerical method can be found in these references.

2.3 Boundary layers modeling

Accretion disc boundary layers have been modeled using the above time dependent method to treat boundary layers around white dwarfs (in Cataclysmic Variable systems, Godon 1995; Godon et al. 1995), pre-main sequence stars (T-Tauriand FUOrionis stars, Godon 1996a), and low mass main sequence stars (Symbiotic AX Per and CI Cyg, Godon 1996b). In these calculations the energy dissipated in the dynamical boundary layer (region over which the angular velocity in the inner part of the disc. decreases from its Keplerian value to adjust to the slowly rotating stellar surface) is radiated radially out ward and heats up a much wider region (the thermal boundary layer, region over which the temperature is significantly larger than the disc temperature). The boundary layer energy is therefore emitted from a rather broad region, and the temperature is consequently rather low.

3. RESULTS

Several models were run for Symbiotic stars and Young Stellar Objects. Preliminary results were published earlier (Godon 1996a & b). 111 the present work we concentrate mainly on the effect of advection in the solutions. Once again, more details on the models and their results can be found in the above references.

We recapitulate the main results of the runs in '1'able 1. The important input parameters are the alpha viscosity parameter and the mass accretion rate. For the symbiotic star $R_* = 0.2R_{\odot}$ and $M_* = 0.5$ M_{\odot} , while for the pre-main sequence star (FU Orionis) $R_* = 4.3R_{\odot}$ and $M_* = M_{\odot}$. The effective temperature in the inner region of tile disc is of the order of ≈ 10 :'1{ in sybmbiotics and ≈ 1041 { for YSOs. The important output parameters are the half thickness of the disc H, the size of the thermal boundary layer (δ_{BL}^{Th}) , the size of the dynamical boundary layer (δ_{BL}^{dyn}) , and the ratio of the advected energy to the accretion energy $\zeta = L_{adv}/L_{acc}$. In the present calculations, the results are exact to only a few percent, due to local oscillations of \dot{M} (Godon 1995).

In Symbiotic stars, advection becomes significant ($\zeta \approx 0.1$) as the mass accretion rate approaches 1 0⁻⁴ M_{\odot}/y (this mass accretion rate is in fact close to the Eddington limit $\dot{M}_{Edd} \approx 2.5 \times 10^{-4} M_{\odot}/y$). The opacity source in Symbiotics is electron scattering which is constant. The advection is therefore mainly dominated by the increase of \dot{M} . This is shown in figure], where we draw all the symbiotic models in the (α , M) plane. Advection does not seems to depend strongly 0.11 α , but rather on M.

In the case of the YSOs, the situation is different. In Table 1 we see that advection appears ($\zeta \approx 0.1$) at high accretion mass rates or for low values of α (here the Eddington

limit is $\dot{M}_{Edd} \approx 5 \times 10^{-3} M_{\odot}/y$). In the non-advective solutions of YSOs the opacity is the free-free and bound-free opacity, function of ρ and T. In these models the density varies by several orders of magnitude ($\rho \approx \dot{M}/\alpha$), while the temperature is not even increased by a factor of 2. As the mass accretion rate increases or the α parameter decreases, the density together with the optical depth increase and eventually the medium cannot cool efficiently. At this stage the central temperature increases and the opacity law changes to free-free and eventually to electron scattering. The medium is then radiation pressure dominated and advection becomes non-negligible. Consequently the advection process is turned on mainly with the increase in the density. In the (α, \dot{M}) plane (shown in figure 2) the lines of constant densities are lines parallel to a diagonal crossing from the lower left corner to the upper right corner. From the graph in figure 2, it can be seen that advection becomes dominant as one moves from a low density (the upper left corner of the plane) to a higher density (the lower right corner).

Furthermore, and the systems calculated here, in which advection is non-negligible in the boundary layer region, are characterized in the inner part of the disc by a non-Keplerian rotation law ($\Omega < \Omega_K$), a vertical thickness $H/r \approx 0.5$, and a luminosity $L < L_{BL}$ (Godon 1996a & b). The inner region of the disc. is consistent with the advection-dominated flow as described by Naraya n & Yi (1994, 1995). The outer part of the disc, however, can be represented by the standard thin disc of Shakura & Sunyaev (1973).

4. D ISCUSSIONS

Since we are not considering a black hole where the energy is swallowed beyond the horizon, we have to consider t he fate of the advected energy. In the present situation the energy is advected into the outer envelop of the accreting star. Prialnik & Livio (1 985) have shown that a fully convective star of mass $0.2M_{\odot}$, accreting matter at a rate $M \approx 10^{-4} M_{\odot}/y$ can increase its radius if it is gaining a significant fraction of the accretion energy $\approx 0.1 L_{acc}$. In the case of the Symbiotics, all the criteria are fulfilled, one can therefore expect the star to increase its radius as the mass accretion rate approaches $10^{-4} M_{\odot}/y$. Indeed, previous calculations of boundary layers in Symbiotics (Godon 1996b) have shown that the boundary layer temperature drop observed in symbiotics near optical maximum (with an estimate increase of \dot{M} from 10-5 to $10^{-4} M_{\odot}/y$) can be explained if the star increases its radius by a factor of 2 or 3. We therefore conclude that the observed boundary layer luminosity drop in symbiotic systems (AX-Per-&-CICyg) is a consequence of the increase of the stellar radius, due to the energy advected in the outer stellar envelop. In YSOs (FU Orionis), it is already commonly believed that the accreting star (initially _a TTauri) increases its radius (from $\approx 2R_{\odot}$ to $\approx 4~R_{\odot}$) as it accretes matter at a high rates. However, calculations similar to those performed by Prialnik & Livio († 985) arc. still missing for pre-main sequence stars with $M_* \approx 1 \times M_{\odot}$.

An additional important property of advection dominated flows is their aptitude to produce bipolar outflows (Narayan & Yi 19 9 4 1995). The Bernoulli parameter is positive in advection dominated flows and violent convections are likely to appear especially close

to the rotation axis. Under these conditions, bipolar outflows can easily form Double jets together with discs are observed in Young Stellar Objects at radio frequencies (see for example the VLA image of HLTau by Rodriguez et al. 1994) and even in the optical (Burrows, St ^aP elfeldt & Watson 1 996). One generally believes the Jets to be produced by the magnetic field of' either the accreting star (the magnetospheric model, see for example Shu et al. 1994a & b, and Najita & Shu 1994) or the -disc -(through -a centrifugally driven wind mechanism as proposed by Blandford & Payne 1982 or through a magnetic pressure explosion as described by Shibata & Ushida 1 985). Magnetic fields are also believed to be responsible for disrupting the inner part of accretion discs, providing therefore an explanation for the inner hole observed in the discs of YSOs (Beckwith et al. 1990 $\,$; Skrutskie et a]. 1990; Edwards et al. 1993; Beckwith 1994). However, recent numerical calcula tions (Godon 1996c) have shown that the inner part of the disc in Trauri stars can be optically thin. This could also explain the inner hole observed in these discs. Therefore, advection dominated accretion flows could also be invoked, at least in sonic systems, as an additional mechanism to produce the observed bipolar outflows. Together with this, advection dominated flows have a substantial sub-Keplerian rotation law, which could significantly spin-down the accreting star at high accretion rate, like in FU Orioni systems, which are believed to be TTauri systems in temporary outbursts. This could explain the slow rotation rate of TTauris stars.

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FIGURE LEGENDS

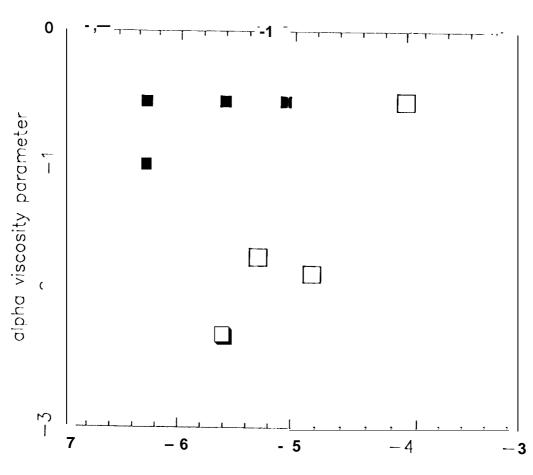
- Figure 1. The models for the Symbiotic. systems are drawn in the (α, M) plane, on a log-log scale. The find circles represent the model for which $\zeta < 0.05$, the empty ones are for the models for which $\zeta > 0.05$. The mass accretion rate is in M_{\odot}/y .
- Figure 2. The models for the FU Orioni systems are drawn in the (α, M) plane, on a log-log scale. The find squares represent the models for which $\zeta < 0.05$, the empty ones are for the models for which $\zeta > 0.05$. The mass accretion rate is in M_{\odot}/y .

TAIL I/I: 1
Advectioninboundary 1,a\'h{\ for various systems}

[1]	[2]	[3]	[4]	[5]	[e]	[7]
System	Ω	Ň	H	δ_{BL}^{Th}	δ_{121}^{dyn}	$\zeta = \frac{L_{adv}}{L_{acc}}$
		(M_{\odot}/y)	(){,)	$(1\{.)$	$(/\{.)$	1.056
Sym	0.05	2s)(-5)	0.17	0.30	0.10	<<1
	0.11	4.()(-5)	0.24 - 0.16	0.:+5	0.15	<
	0.14	1.0(-5)	0.15 - 0.12	0.40	0.05	<< 1
	0.07	4.0(-5)	0.30 - 0.20	0.70	0.20	<< 1
	0.1	7.0(-5)	0.35-0.20	1.00	0.20	< < 1
	0.22	9.0(-5)	0.26.(1.20	0.70	0.20	0.07
	0.18	1 .8(-4)	0.37-0.23	0.70	0.40	0.11
	0.1G	3.0(-4)	0.40 - 0.25	0.70	0.50	(1.12
YSO	0.3	5.()(-7)	[1,10	0.30	0.10	<< 1
	0.1	5.0(-7)	0.05 - 0.12	0.40	0.10	< < 1
	0.3	2.5(-G)	[1.1]	0.3(1	0.10	<< 1
	0.06	5.()(-G)	0.12	0.50	0.10	< 1
	0.3	8.5(-6)	0.12	0.50	0.10	< < 1
	0.005	2.5(-6)	0.25	***	0.50	0.14
	0.02	5.0(-6)	0.26		0.50	0.23
	0.015	1.5(-5)	0.30	•••	0.40	0.21
	0.3	1.0(-4)	0.40	•••	0.50	0.27

Note: In column 4: there are two values for H, the first value corresponds to the vertical thickness in the boundary layer region, the second one refers to the vertical thickness in the disc. In column 5: no distinct thermal boundary layer component is observed in the solutions of the advective YSOs (lower set of results). Incolumn 7: for values of ζ of a few percents ($\zeta < 0.(15)$, we wrote $\zeta << 1$, since the calculations are exact to only a few percents due to local oscillations of M.

Pre Main Sequence Systems



Mass accretion rate

Low Mciss Main Sequence Systems

